Almost Optimal Distance Oracles for Planar Graphs

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Preprocess an \( n \)-vertex planar graph \( G = (V, E) \) with nonnegative arc lengths, so that given any \( u, v \in V \) we can compute \( d(u, v) \) efficiently.
Problem definition

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We want:

- Fast queries, ideally $Q = O(1)$.
- Small size, ideally $S = O(n)$.
- Fast construction, ideally $T = O(n)$.

The most important tradeoff is between query-time $Q$ and size $S$. 
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The most important tradeoff is between query-time $Q$ and size $S$. 
Previous work

The tradeoff between the query-time $Q$ and the size $S$ of the structure:

\[
\frac{\lg Q}{\lg n} \quad \frac{\lg S}{\lg n}
\]

Our result

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Previous work

The tradeoff between the query-time $Q$ and the size $S$ of the structure:

$$\frac{\lg Q}{\lg n} = \frac{1}{4} \quad \frac{\lg S}{\lg n}$$

Djidjev and Arikati et al. achieved $Q = O(n^2/S)$. 
Previous work
The tradeoff between the query-time $Q$ and the size $S$ of the structure:

\[
\frac{\log Q}{\log n} = \frac{4}{3} - \frac{2}{3} \cdot \frac{\log S}{\log n}
\]

Fakcharoenphol and Rao showed that $S = \tilde{O}(n)$ and $Q = \tilde{O}(\sqrt{n})$ is possible.
Previous work

The tradeoff between the query-time $Q$ and the size $S$ of the structure:

\[
\frac{\lg Q}{\lg n} = \frac{4}{3}, \quad \frac{2}{3}, \quad \frac{1}{2}, \quad \frac{1}{4}, \quad \frac{1}{3}
\]

\[
\frac{\lg S}{\lg n} = \frac{5}{3}, \quad \frac{3}{2}, \quad 1, \quad \frac{4}{3}, \quad \frac{5}{3}, \quad 2
\]

This has been extended to $Q = \tilde{O}(n/\sqrt{S})$ for essentially the whole range of $S$ in a series of papers.
Previous work

The tradeoff between the query-time $Q$ and the size $S$ of the structure:

\[
\frac{\log Q}{\log n} \quad \frac{\log S}{\log n}
\]

In 2017, Cohen-Addad, Dahlgaard, and Wulff-Nilsen showed that this is not optimal, and $S = O(n^{5/3})$ with $Q = O(\log n)$ is possible.
Previous work

The tradeoff between the query-time $Q$ and the size $S$ of the structure:

$\frac{\lg Q}{\lg n}$ vs $\frac{\lg S}{\lg n}$

In 2018, Gawrychowski et al. improved this to $S = O(n^{1.5})$ and $Q = O(\log n)$.
Previous work
The tradeoff between the query-time $Q$ and the size $S$ of the structure:

$$\frac{\lg Q}{\lg n}$$

$$\frac{\lg S}{\lg n}$$

We improve this to $S = O(n^{1+\epsilon})$ and $Q = \tilde{O}(1)$ for any $\epsilon > 0$. 

Our result
Main result

We show the following tradeoffs for $\langle$space, query-time$\rangle$:

1. $\langle \tilde{O}(n^{1+\epsilon}), O(\log^{1/\epsilon} n) \rangle$, for any constant $1/2 \geq \epsilon > 0$;
2. $\langle O(n \log^{2+1/\epsilon} n), \tilde{O}(n^\epsilon) \rangle$, for any constant $\epsilon > 0$;
3. $\langle n^{1+o(1)}, n^{o(1)} \rangle$.

The oracle is based on a recursive view of the point location mechanism for Voronoi diagrams of Gawrychowski et al. [SODA’18].
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There always exists a Jordan curve separator of size $O(\sqrt{n})$ such that there are at most $\frac{2}{3} n$ vertices on its inside/outside.
Cycle Separators

Miller [JCSS’86]

There always exists a Jordan curve separator of size $O(\sqrt{n})$ such that there are at most $\frac{2}{3} n$ vertices on its inside/outside.
$r$-divisions

For $r \in [1, n]$, a decomposition of the graph into:

- $O(n/r)$ pieces;
- each piece has $O(r)$ vertices;
- each piece has $O(\sqrt{r})$ boundary vertices (vertices incident to edges in other pieces).

We denote the boundary of a piece $P$ by $\partial P$ and assume that all such vertices lie on a single face of $P$. 
Recursive $r$-divisions

For $r_1 < r_2 < \cdots < r_m \in [1, n]$, we can efficiently compute $r_i$-divisions, such that each $r_i$-division respects the $r_{i+1}$-division.
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Multiple Source Shortest Paths (MSSP)

Klein [SODA’05]

There exists a data structure requiring $O(n \log n)$ space that can report in $O(\log n)$ time the distance between any vertex on the infinite face (boundary vertex) and any vertex in the graph.
Warm-up: Space $\tilde{O}(n^{4/3})$, Query-time $\tilde{O}(n^{1/3})$

- Compute an $r$-division.
- For each vertex $u \in P$, store additive weights $d_G(u, p)$ for $p \in \partial P$. 
  Space $O(n \cdot \sqrt{r})$.
- For each piece $P$, store an MSSP data structure for the outside of $P$ 
  with sources $\partial P$. 
  Space $\tilde{O}(n/r \cdot n)$. 
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We decompose the path on the last boundary vertex it visits.
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At query, find vertex $s \in \partial P$, minimizing $d_G(u, s) + d_{G\setminus(P\setminus\partial P)}(s, v)$. This is called point location.
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Perform point location by trying all $O(\sqrt{r})$ boundary vertices.
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First goal

Instead of trying all possible \( O(\sqrt{r}) = O(n^{1/3}) \) candidate boundary vertices, we want to compute the last boundary vertex \( s \) visited by the shortest path in \( \tilde{O}(1) \) time.
Point location

- Each vertex $u$ defines a set of additive weights $d_G(u, p)$ for $p \in \partial P$.

Gawrychowski et. al. [SODA’18]

Given an MSSP data structure for the outside of $P$, with sources $\partial P$, there exists an $\tilde{O}(|\partial P|)$-sized data structure for each set of additive weights for $\partial P$ that answers point location queries in $\tilde{O}(1)$ time.
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Warm-up 2: Space $\tilde{O}(n^{4/3})$, Query-time $\tilde{O}(1)$

- Compute an $r$-division.
- For each vertex $u \in P$, store additive weights $d_G(u, p)$ for $p \in \partial P$. Preprocess these for point location.
  Space $O(n \cdot \sqrt{r})$.
- For each piece $P$, store an MSSP data structure for the outside of $P$ with sources $\partial P$.
  Space $\tilde{O}(n/r \cdot n)$.

At query, perform point location by trying all possible $O(\sqrt{r})$ candidate boundary vertices.
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At query, perform point location in $\tilde{O}(1)$ time!
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Second goal

Shrink pieces so that cost for storing additive weights shrinks.

- Compute an $n^\epsilon$-division.
- For each vertex $u \in P$, store additive weights $d_G(u, p)$ for $p \in \partial P$. Preprocess these for point location.
- For each piece $P$, store the required information to support:
  - distance queries from $\partial P$ to vertices outside $P$;
  - point location.
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The Voronoi cell of each site consists of all vertices closer to it with respect to the additive distances.
Because all sites are adjacent to one face, the diagram can be described by a tree on $O(|\partial P|) = O(\sqrt{r})$ vertices (independent of $n$).
Any tree on $k$ vertices contains a centroid vertex $x$ such that every component of $T \setminus \{x\}$ is of size $\frac{2}{3}k$. 
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Main idea: Consider the centroid. Find which subtree contains edges adjacent to the Voronoi cell containing $v$. Recurse.
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The centroid vertex corresponds to a trichromatic face of the original graph.
Find which \( s_k \) among \( s_1, s_2, s_3 \) is closest to \( v \).
(distance queries from boundary vertices – MSSP)

Check whether \( v \) is left or right of \( p_k \). (left/right query – MSSP)
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Idea

Recurse, exploiting the structure of such queries.

Find which $s_k$ among $s_1, s_2, s_3$ is closest to $v$.
(distance query from boundary vertices)
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

- Compute a recursive $r$-division for $r_i = n^{i\cdot\epsilon}$.
- For each piece $P$ of the $n^\epsilon$-division, for each vertex $u \in P$, store a Voronoi diagram for the outside of $P$ with sites $\partial P$ and additive weight $d_G(u, p)$ for $p \in \partial P$. Space $\tilde{O}(n \cdot \sqrt{r_1})$.
- For each piece $P$, store the required information to answer distance queries from $\partial P$ to vertices outside $P$.

At query, perform point location.
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We can not afford to store an $\Omega(n)$-sized MSSP for each of the $n^{1-\epsilon}$ pieces.
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

Store an MSSP for piece $Q$ of the $n^{2\cdot\epsilon}$-division that contains $P$. This handles the case $v \in Q$.

Space: $\tilde{O}(n^{1-\epsilon} \cdot n^{2\epsilon}) = \tilde{O}(n^{1+\epsilon})$. 
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Case $v \notin Q$: each $p \in \partial P$ stores a Voronoi diagram for the outside of $Q$ with sites $\partial Q$.
Space: $\tilde{O}(n^{1-\epsilon} \cdot n^{\epsilon/2} \cdot n^{2\epsilon/2}) = \tilde{O}(n^{1+\epsilon/2})$. 
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

Repeat the same reasoning for increasingly larger pieces of sizes $n^{i\cdot\epsilon}$, for $i = 1, \ldots, 1/\epsilon$. There are $n^{1-i\epsilon}$ pieces at level $i$, each stores MSSP and Voronoi diagrams of size $\tilde{O}(n^{(i+1)\epsilon})$. Total space: $\tilde{O}(\frac{1}{\epsilon} n^{1+\epsilon})$. 
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

Smaller pieces share the MSSP data structures at higher levels.
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

Each point location query, either gets answered at the current level, or reduces to $O(\log n)$ point location queries at a higher level.
Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

If not earlier, then in the top level we answer the point location query in $O(\log^2 n)$ time.
Query time: $O(\log^{1/\epsilon} n)$. 
Tradeoffs and construction time

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Issues and extras:
- left/right queries;
- \( \partial P \) is not a single face of \( P \) (holes);
- constructing these oracles in \( O(n^{3/2+\epsilon}) \) time.
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Open problems

- Can we get $\tilde{O}(n)$ space and $\tilde{O}(1)$ query time?

- Can we get the construction time to be $\tilde{O}(n)$?

- Improvements on dynamic distance oracles?
  Currently:
  1. exact: UB $\tilde{O}(n^{2/3})$ ; LB $\tilde{O}(n^{1/2})$ (conditioned on APSP)
  2. approx.: UB $\tilde{O}(n^{1/2})$ (undirected) ; no LB.
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Thanks! Questions?