Pattern Matching with Mismatches and Wildcards

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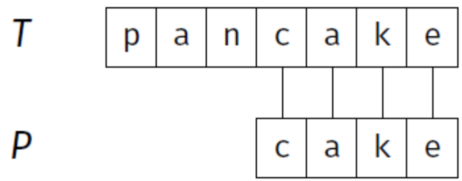
ESA 2024

The Problem

Pattern Matching

Given a text T and a pattern P, compute the occurrences of P in T.



Easy; a linear-time algorithm is known since 1970 [Morris-Pratt]. However, looking for exact matches of P in T might be too restrictive: think of spelling mistakes and corrupt data.

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Pattern Matching with Wildcards

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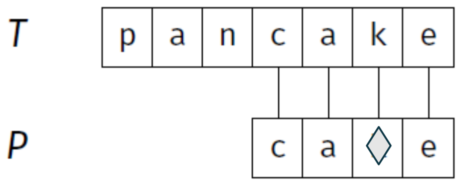

If we know the corrupt positions, we can replace their entries with wildcards ($\lozenge$) which match all letters of the alphabet and perform exact pattern matching. A long series of works has culminated in an elegant FFT-based $\mathcal{O}(|T| \log |P|)$-time algorithm [Clifford–Clifford; 2007].

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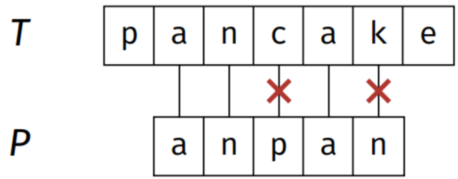
Pattern Matching with Mismatches

Given a text T, a pattern P, and an integer threshold k, compute the substrings of T that are at Hamming distance at most k from P.



Alternatively, we can look for substrings of T that are close to P, e.g., under the Hamming distance. This is a much harder problem; it admits an $\tilde{\mathcal{O}}(|T| + k \cdot |T|/\sqrt{|P|})$-time solution [Gawrychowski–Uznanski; 2018].

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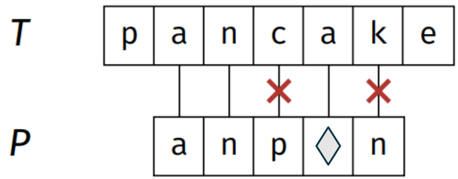

In this work, we revisit the variant of problem where some of the corrupt positions are known.

Previous Work and Our Result

$n = |T|, m = |P|$

$D = \#$ wildcards $= 9$

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$\tilde{\mathcal{O}}(n\sqrt{k} + n \cdot \min\{\sqrt[3]{Gk}, \sqrt{G}\})$ [Nicolae-Rajasekaran; 2017]

$\mathcal{O}(n + (n/m)(D+k)(G+k))$ [this work]

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Fast when D, G, and k are small relative to n. For $m = n/2$, $k = G = n^{2/5}$, and $D = n^{3/5}$, our algorithm takes $\mathcal{O}(n)$ time, improving over $\mathcal{O}(n^{6/5})$.

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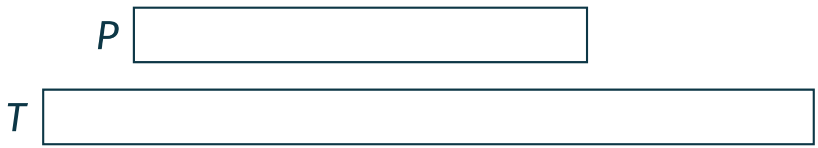

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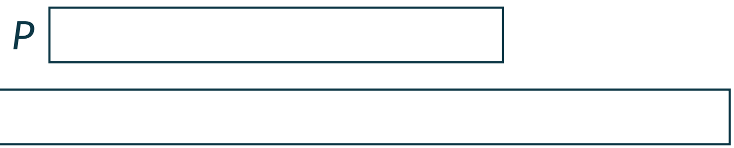

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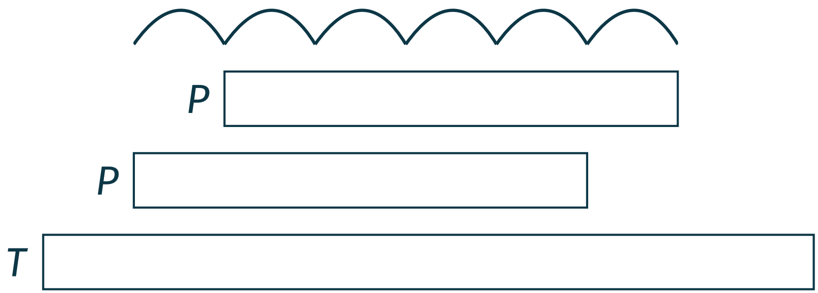

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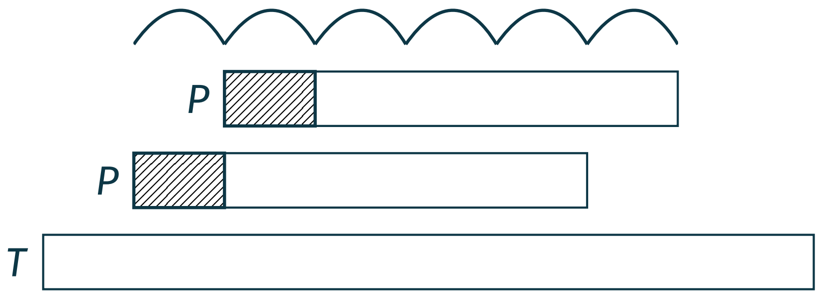

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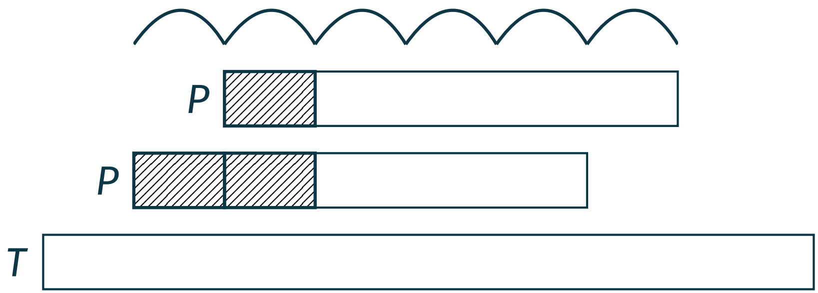

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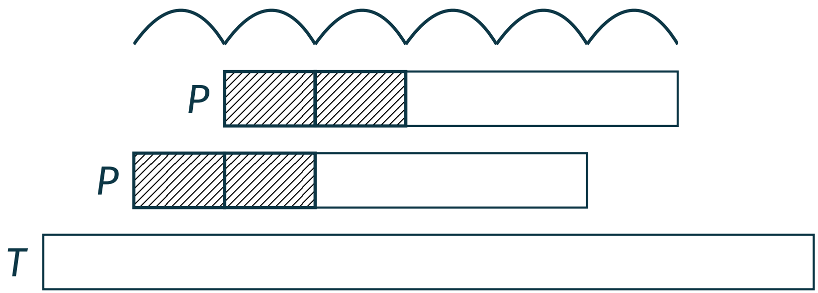

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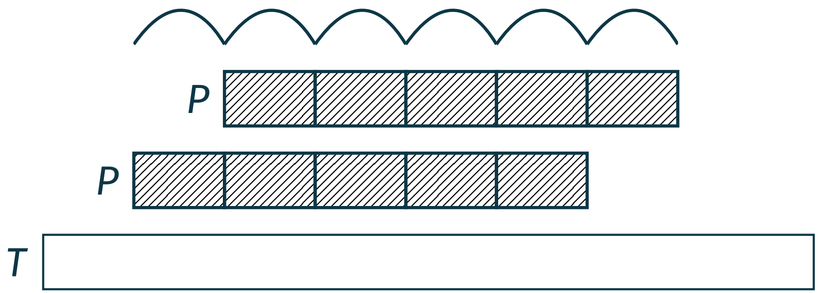

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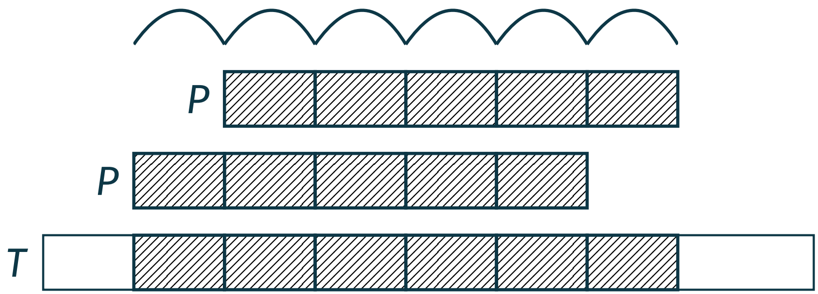

The fragment of T spanned by P's occurrences is periodic as well.

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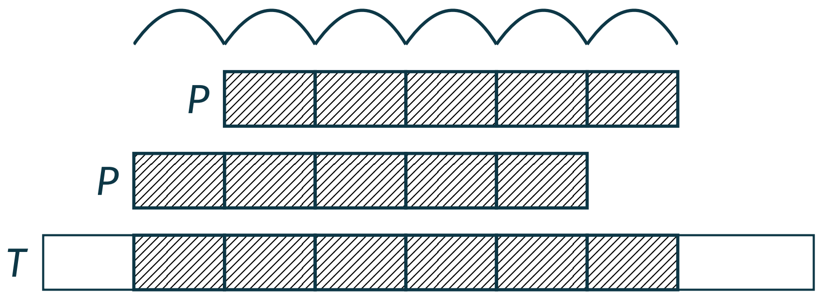

The standard trick: Our assumption on the length of the text is not restrictive. If the text is much longer that the pattern, we can always consider separately $\mathcal{O}(n/m)$ fragments of T of length $\leq 3/2\, m$ that overlap by $m - 1$ positions.

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This structural insight leads to an alternative $\mathcal{O}(n + k^2)$-time algorithm [CKW'20].

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Bonus: A simple $\mathcal{O}(n + DG)$-time algorithm for exact PM with wildcards.

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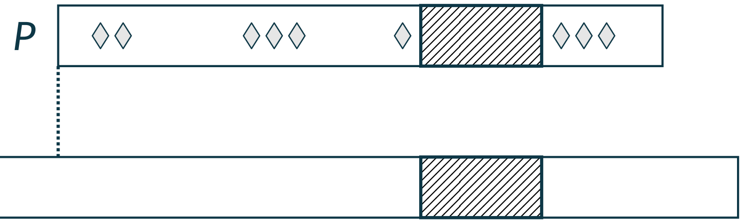

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Observation: If the chunk C is aperiodic, its occurrences cannot overlap by more than $|C|/2$ positions $\Rightarrow$ at most $n/(|C|/2) = \mathcal{O}(G \cdot n/m) = \mathcal{O}(G)$ occurrences.

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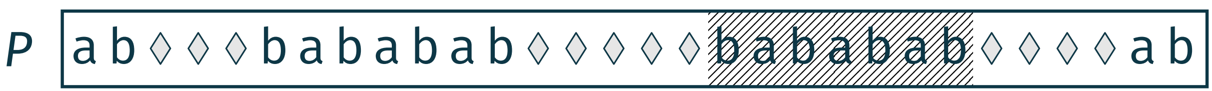

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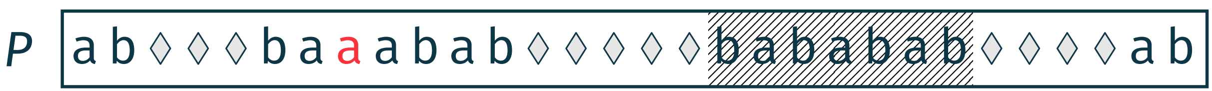
- Else, we have to work a bit more. :)



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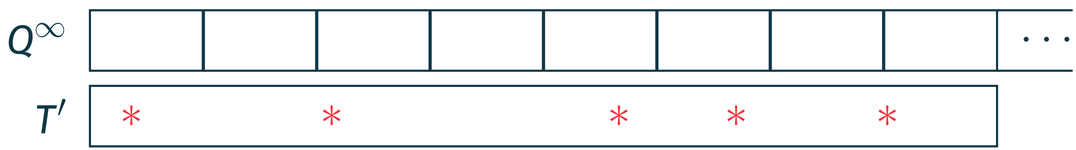

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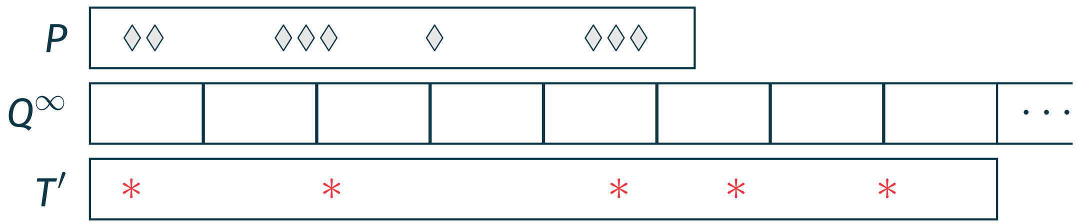

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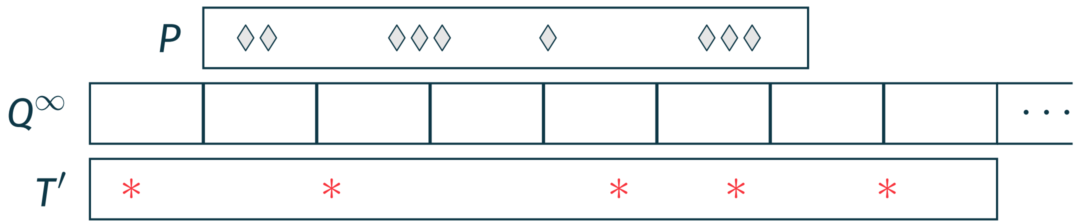

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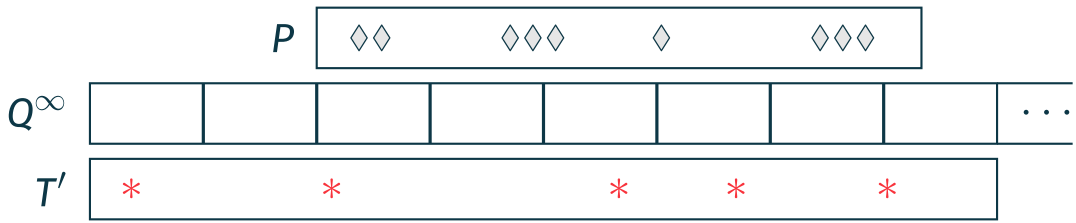

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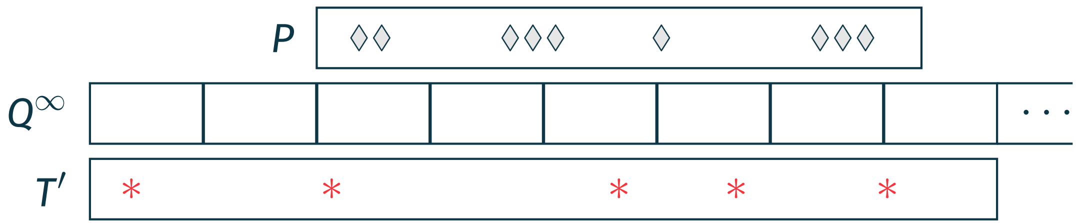

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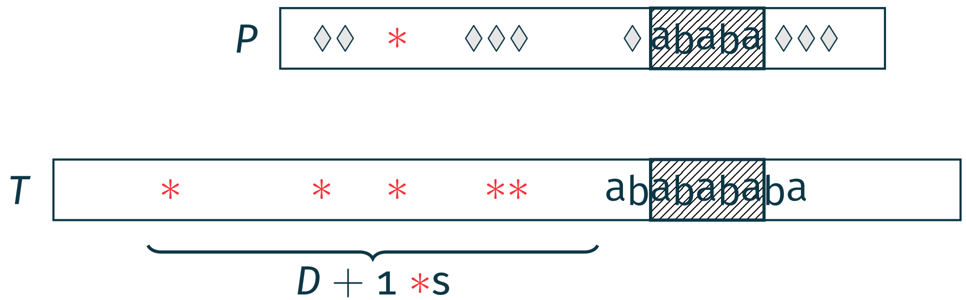

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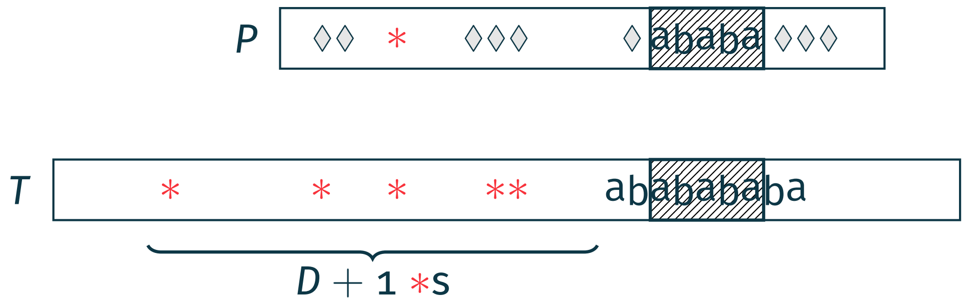

Observation: The misperiod in P must be aligned with one of the first $D+1$ misperiods in T. We thus have $\mathcal{O}(D)$ candidates, and each can be verified in $\mathcal{O}(G)$ time. Total time: $\mathcal{O}(DG^2)$.

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A periodicity-based argument yields that we now need to verify $\mathcal{O}(D)$ candidates over all C-runs! Total time: $\mathcal{O}(DG)$.

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Lemma: Let V be a binary vector of size N with $M := \|V\|$ 1s. We can efficiently compute a large set $U \subseteq [1\,..\,N]$ such that for each $i \in U$ and radius $r \in [1\,..\,N]$, $\|B_V(i, r)\| \leq 8r \cdot M/N$.

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We simply apply the above lemma with $\Diamond$s mapped to 1s and other letters mapped to 0s and then select the chunk so that it contains a position in U. We call such positions sparsifiers.

What about mismatches?

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We open the black-box of [CKW'20], ensure that some of the considered substrings contain sparsifiers, and refine the analysis.

Lower Bound on the Arithmetic Progressions

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We use such sets to construct P and T such that P has $\Omega((D+k)\cdot(k+1))$ k-mismatch occurrences in T and no three occurrences form an arithmetic progression.

Final Remarks

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Open problems:

- Is the algorithm optimal?
- Close the gap on the number of arithmetic progressions.
- Edit distance instead of Hamming?
- More applications for sparsifiers?

The End

Thank you for your attention!

Questions?