Dynamic String Alignment

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$$\begin{array}{ccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
S = & \mathtt{a} & - & \mathtt{b} & \mathtt{a} & \mathtt{a} & \mathtt{b} & \mathtt{a} & \mathtt{c} & \mathtt{b} & \mathtt{b} \\
 & | & & | & | & | & & & | & \cdot & | \\
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Alignment's weight: $6w_{match} + w_{mis} + 3w_{gap}$.

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Related Work

There is a textbook $\mathcal{O}(n^2)$-time dynamic programming algorithm.

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A strongly subquadratic-time algorithm would refute the Strong Exponential Time Hypothesis (SETH).

[Backurs-Indyk; SIAM Journal on Computing 2018]

[Bringmann-Künnemann; FOCS 2015]

[Abboud-Hansen-Vassilevska Williams-Williams; STOC 2016]

Related Work

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Several works considered prepending letters or deleting the first letter in either of the strings, culminating in an $\mathcal{O}(n)$-time algorithm.

[Landau-Myers-Schmidt; SIAM Journal on Computing 1998]

[Kim-Park; Journal of Discrete Algorithms 2004]

[Ishida-Inenaga-Shinohara-Takeda; FCT 2005]

[Tiskin; arxiv 2007]

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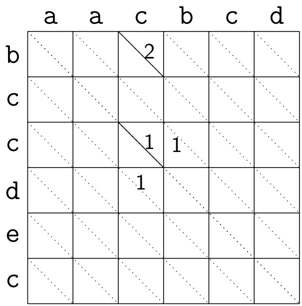
- Based on black-boxes from planar graphs.

Preliminaries



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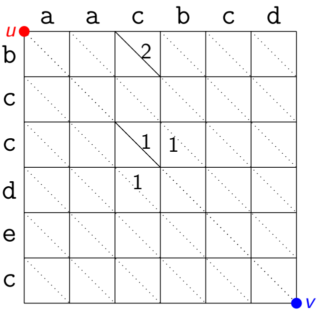

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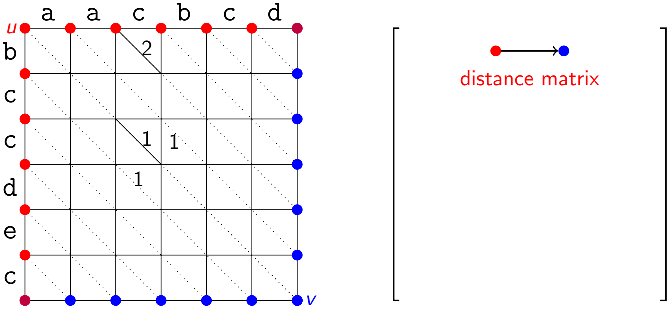

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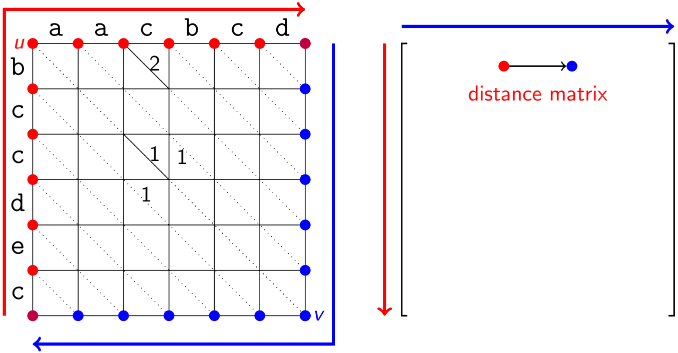

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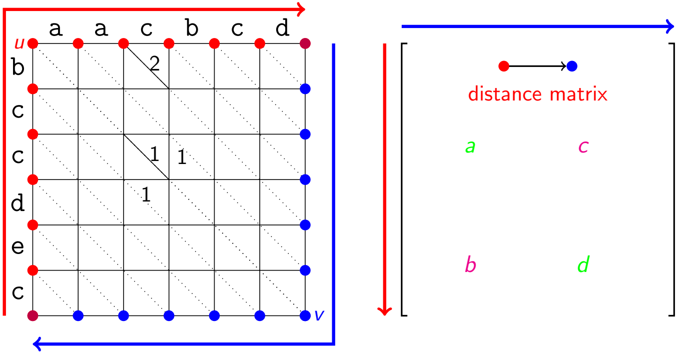

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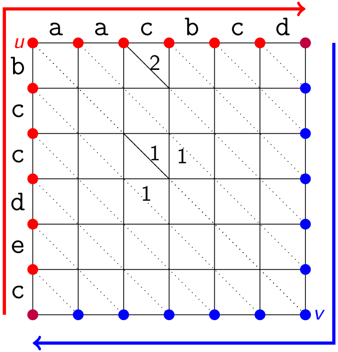

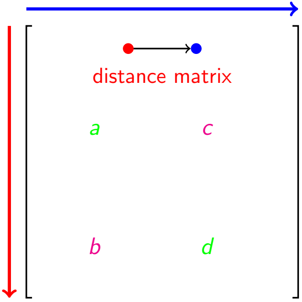

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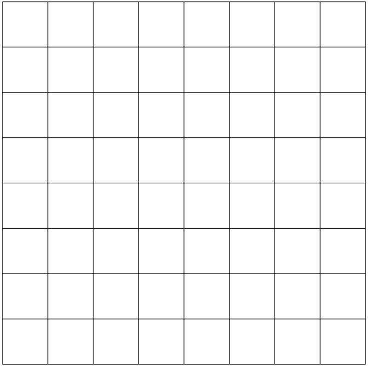
This distance matrix is Monge, and, in fact, unit-Monge.

Unit-Monge Matrices

M



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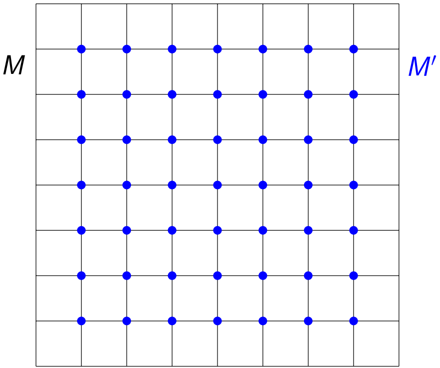

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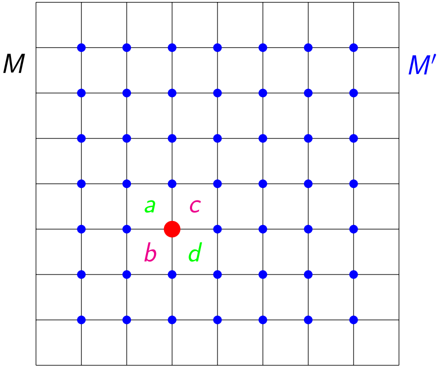

● $= b + c - a - d$

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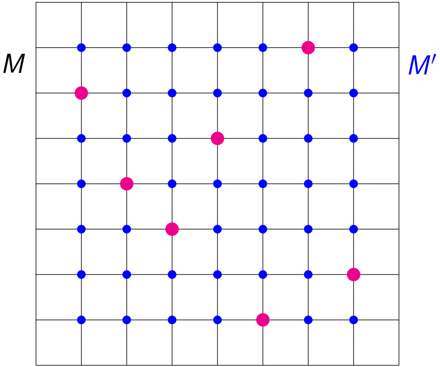

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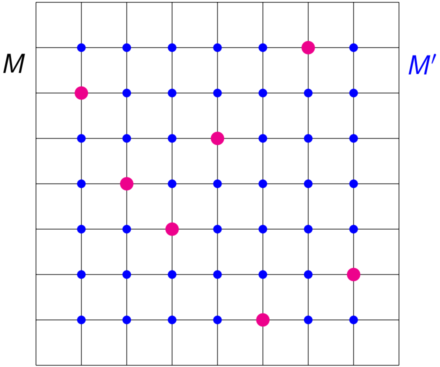

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We can represent an $n \times n$ unit-Monge matrix M in $\tilde{\mathcal{O}}(n)$ space so that each entry can be retrieved in $\tilde{\mathcal{O}}(1)$ time.

Distance Product of Unit-Monge Matrices

Distance Product

The $(\min, +)$ product or distance product of an $m \times k$ matrix A and a $k \times n$ matrix B, denoted by $A \odot B$ is an $m \times n$ matrix C, such that $C[i,j] = \min_{1 \le r \le k} \{A[i,r] + B[r,j]\}$.

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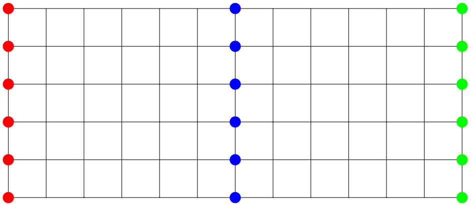

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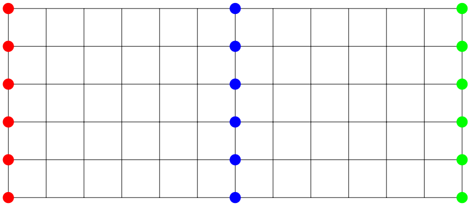

$A :=$ distance matrix (red → blue)

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Efficient $(\min, +)$ multiplication [Tiskin; Algorithmica 2015]

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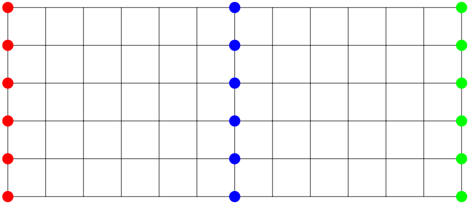

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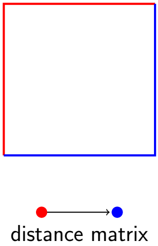

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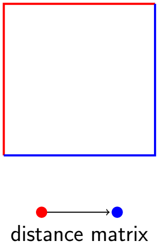

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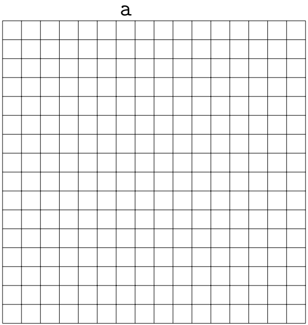

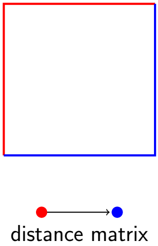

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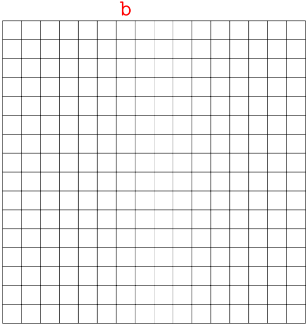

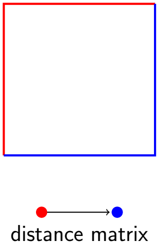

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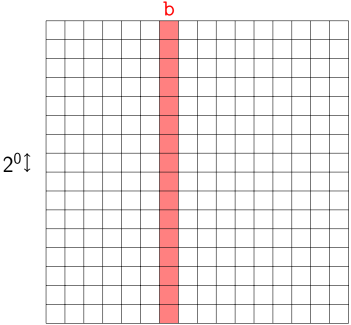

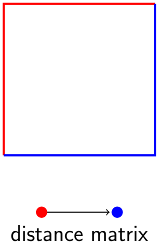

$\mathcal{O}(n/2^i)$ distance matrices change at level i.

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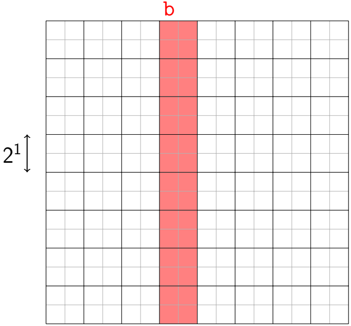

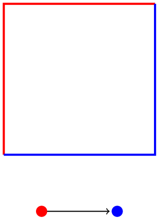

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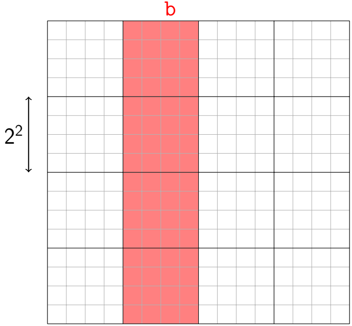

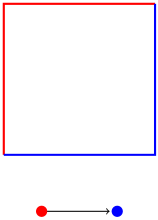

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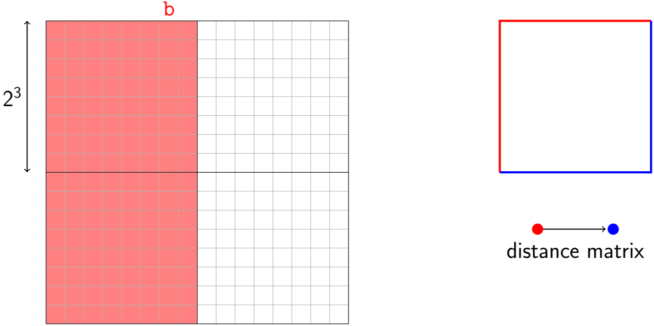

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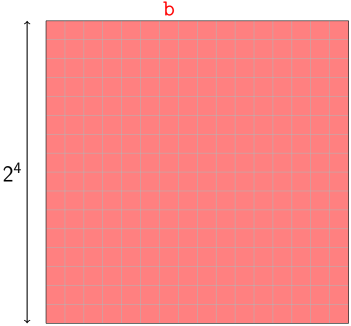

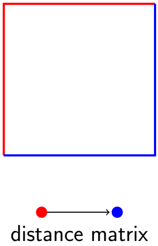

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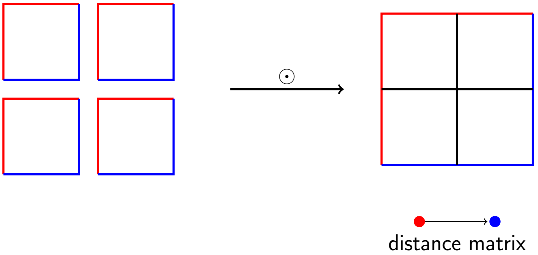

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Each of them is recomputed from four distance matrices of the previous level in $\mathcal{O}(2^i \log(2^i))$ time using distance multiplication.

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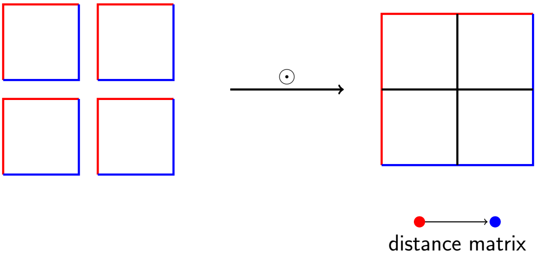

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Each of them is recomputed from four distance matrices of the previous level in $\mathcal{O}(2^i \log(2^i))$ time using distance multiplication.

The total update time is thus $\mathcal{O}(n \log^2 n)$.

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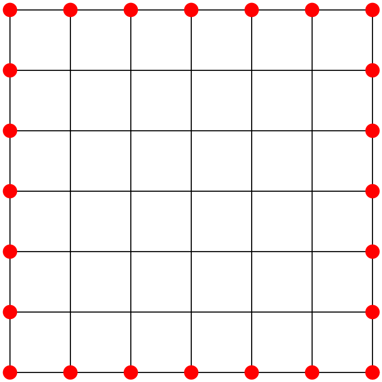
Next: An $\tilde{\mathcal{O}}(n\sqrt{n})$-time algorithm for integer weights of size $n^{\mathcal{O}(1)}$ using techniques for computing shortest paths in planar graphs.

MSSP

Multiple Source Shortest Paths (MSSP) [Klein; SODA 2005]

We can construct in nearly-linear time (in the size of the graph) a data structure that can report in logarithmic time the distance between any node on the infinite face and any node in the graph.



FR-Dijkstra

Dense Distance Graphs

The distance matrix capturing pairwise distances between the vertices of a set ∂H of vertices of a planar graph H, lying on a single face, can be computed in $\tilde{\mathcal{O}}(|H| + |\partial H|^2)$ time using MSSP.

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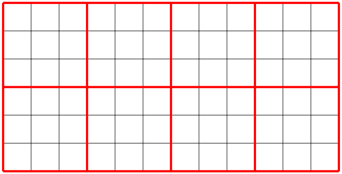

Before: distance product. Now: SSSP computations, many DDGs.

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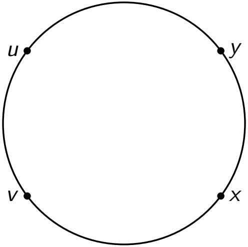

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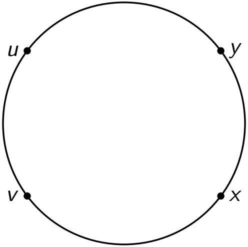

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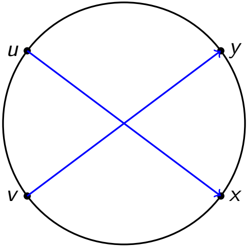

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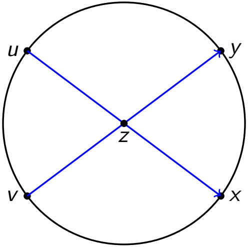

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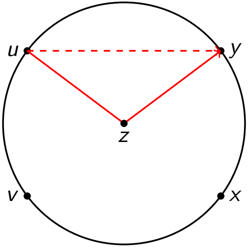

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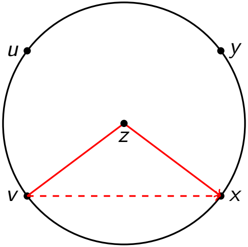

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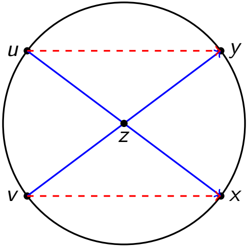

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Algorithm for Large Weights

	a	a	c	b	c	d	d	a	a	e	a	d
a												
b												
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c												
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d												
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c												

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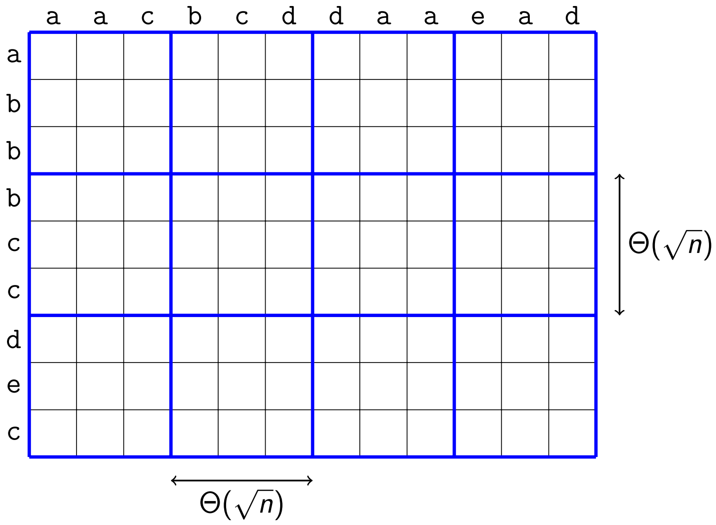

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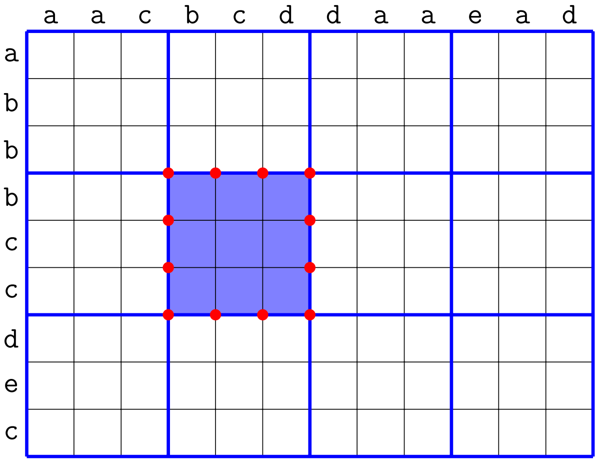

We maintain a DDG for each piece P with the set of "boundary" vertices as ∂P. $|P| = \Theta(n)$, $|\partial P| = \Theta(\sqrt{n})$.

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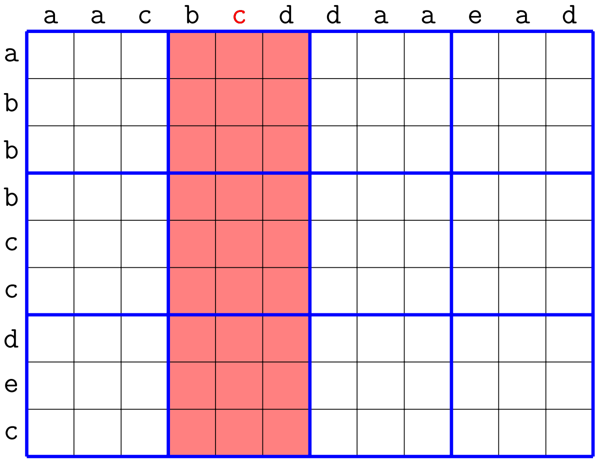

Each update in one of the strings affects $\mathcal{O}(\sqrt{n})$ pieces. The DDG information for each piece is recomputed in $\tilde{\mathcal{O}}(n)$ time using MSSP.

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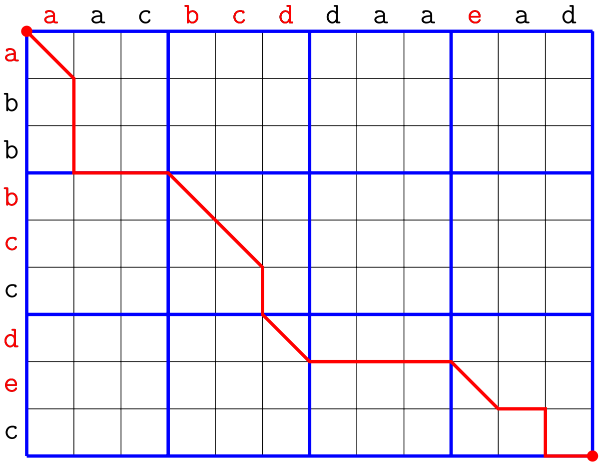

We run FR-Dijkstra on the union of $\mathcal{O}(\sqrt{n} \cdot \sqrt{n}) = \mathcal{O}(n)$ DDGs.
The runtime is $\tilde{\mathcal{O}}(n\sqrt{n})$, since each DDG has $\mathcal{O}(\sqrt{n})$ vertices.

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Thank you for your attention!